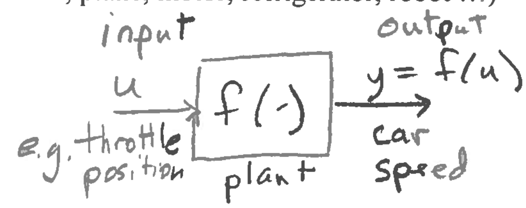
Mechanical Systems Laboratory
Basic Control Concepts; Example of Feedback Control of Op Amp

1. Basic Control Concepts

a. The problem of automatic control

Given a system with inputs and outputs (the "plant" – e.g. a car, plane, motor, refrigerator, robot …)
And a desired output y_d
Find an input u to give you the desired output y_d

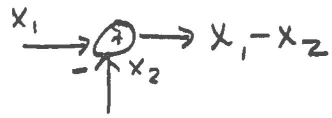


b. Block diagrams

Useful notation for visualizing control systems.
Two common blocks:

1) Gain block

2) Summer block

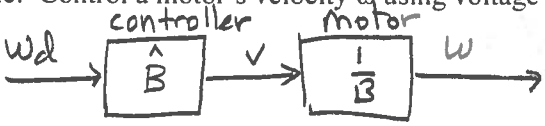


c. Two general approaches to designing the input "u"

Approach 1: Feedforward Control (or "Open Loop" Control)

Choose input to plant based on knowledge of plant (i.e. based on an "inverse model" of the plant).

Example: Control a motor's velocity ω using voltage v as the input. What should v be such that $\omega = \omega_d$?



$V = \hat{B}\omega_d$

$\omega = \frac{V}{B} = \frac{\hat{B}}{B}\omega_d$

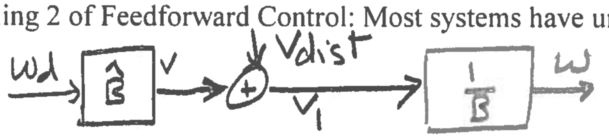
$\wedge$ = an estimate

If $\hat{B} = B$, $\omega = \omega_d$

Shortcoming 1 of Feedforward Control: Need to have an accurate model of the plant

If $\hat{B} = .5B$, then $\omega = .5\omega_d$

Shortcoming 2 of Feedforward Control: Most systems have unpredicted "disturbances" that affect the output



$\omega = \frac{V_1}{B} = \frac{V + V_{dist}}{B}$

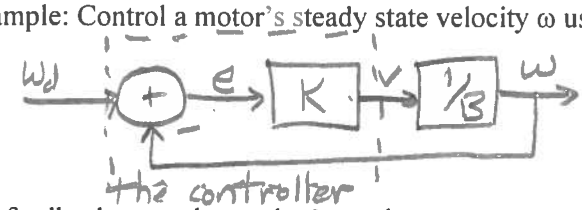
$\omega = \frac{\hat{B}}{B}\omega_d + \frac{1}{B}V_{dist}$

If V_{dist} is big, $\omega \neq \omega_d$

Approach 2: Feedback Control (or "Closed Loop" Control)

Refers to measuring a system's output and "feeding it back" to change the input so $y = y_d$.
Also called "closed-loop control" because you "close the control loop" by "feeding back" the sensed output.
Usually, you subtract the system's output from the desired output to get an error signal, then apply an input to the system proportional to the error in the direction that reduces the error (negative feedback).

Example: Control a motor's steady state velocity ω using voltage v as the input.



$e = \omega_d - \omega$

$V = Ke = K(\omega_d - \omega)$

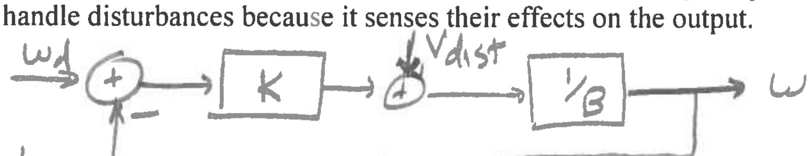
$\omega = \frac{1}{B}V = \frac{K}{B}(\omega_d - \omega)$

$\omega\left(1 + \frac{K}{B}\right) = \frac{K}{B}\omega_d$

$$\boxed{\omega = \frac{K}{B+K}\omega_d}$$

For feedback control, you don't need an accurate model of the plant (you just need to know which way to "push" the plant to reduce error). Feedback control can also handle disturbances because it senses their effects on the output.

If $K >> B$, then $\omega = \omega_d$



Solve: $\omega = \frac{K}{K+B}\omega_d + \frac{1}{K+B}V_{dist}$ if $K >> B$, $\omega \approx \omega_d$

What are some shortcomings of feedback control?

1) an error has to develop before the controller acts
2) delay causes instability
3) requires a sensor

Many control systems (like the human neural movement control system) use feedforward and feedback control together, as they complement each other well.

2. Example: Feedback Control of an Op Amp

Op-amps are often used with a negative feedback configuration. In fact, this configuration let's us assume "Golden Rule" 2 in our analysis …

Golden Rules of Op-amp Circuit design:

1. Input currents are zero (op amps are designed to have a high input resistance)
2. Input voltages are equal (if operating in linear region, and connected with negative feedback)

Let's see if we can derive Golden Rule 2. Consider the voltage following circuit:

This circuit is useful if we want to connect two different circuits together without having the connection change the way the individual circuits operate. That is, the voltage follower circuit can serve as "buffer" to isolate different parts of a more complex circuit. This is because i+ = (no current draw) and Vout = Vin. It's as if we are grabbing a voltage from a circuit and just letting it drive another circuit, without fear of it loading the first circuit.

But why does Vout = Vin? That is, why is Golden Rule 2 true?

Start with the fundamental operating characteristic of an op amp:

$$V_{out} = K(V_+ - V_-) \quad K \text{ is big by design}$$

Now, because of the negative feedback, there is a wire connecting Vout to V-, so:

$$V_- = V_{out}$$

Note also that Vin is connected to V+ by a wire, so:

$$V_+ = V_{in}$$

Using these relationships we find that:

$$V_{out} = K(V_{in} - V_{out})$$

Solving for Vout

$$V_{out}(1+K) = KV_{in}$$

$$V_{out} = \frac{K}{1+K} V_{in}$$

If K is very big, then Vout = Vin. So Golden Rule 2 is true because the op amp is measuring its output, comparing it to Vin, and changing its output very quickly so the two match each other.